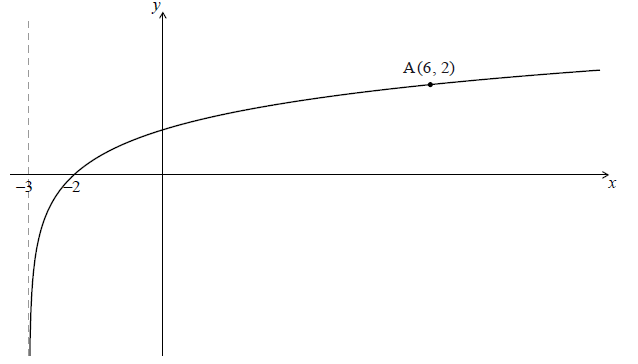
# Chp 3 & 4 Questions

**1a.** *[4 marks]*

Let  for  . Part of the graph of *f* is shown below.



The graph passes through A(6, 2) , has an *x*-intercept at (−2, 0) and has an asymptote at  .

Find *p* .

**1b.** *[5 marks]*

The graph of *f* is reflected in the line  to give the graph of *g* .

(i) Write down the *y*-intercept of the graph of *g* .

(ii) Sketch the graph of *g* , noting clearly any asymptotes and the image of A.

**1c.** *[4 marks]*

The graph of  is reflected in the line  to give the graph of  .

Find  .

**2a.** *[1 mark]*

Jose takes medication. After *t* minutes, the concentration of medication left in his bloodstream is given by  , where *A* is in milligrams per litre.

Write down  .

**2b.** *[2 marks]*

Find the concentration of medication left in his bloodstream after 50 minutes.

**2c.** *[5 marks]*

At 13:00, when there is no medication in Jose’s bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again?

**3a.** *[2 marks]*

Let  , for  .

Show that  .

**3b.** *[3 marks]*

Find the value of  and of  .

**3c.** *[6 marks]*

The function *f* can also be written in the form  .

(i) Write down the value of *a* and of *b* .

(ii) Hence on graph paper, **sketch** the graph of *f* , for  ,  , using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

**3d.** *[1 mark]*

Write down the value of  .

**3e.** *[4 marks]*

The point A lies on the graph of *f* . At A,  .

On your diagram, sketch the graph of  , noting clearly the image of point A.

**4a.** *[4 marks]*

Let  and  .

Express  in the form  , where  .

**4b.** *[3 marks]*

The graph of *g* is a transformation of the graph of *f* . Give a full geometric description of this transformation.

**5a.** *[3 marks]*

Find the value of  .

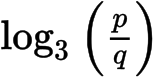
**5b.** *[4 marks]*

Find the value of  .

**6a.** *[7 marks]*

Let  and  .

(a) Find  .

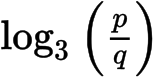
(b) Find  .

(c) Find  .

**6b.** *[2 marks]*

Find  .

**6c.** *[2 marks]*

Find  .

**6d.** *[3 marks]*

Find  .

**7a.** *[7 marks]*

Let  and  , where  , and  . Let  be the region enclosed by the -axis, the graph of  , and the graph of  .

Let .

(i) Sketch the graphs of  and  on the same axes.

(ii) Find the area of  .

**7b.** *[5 marks]*

Find the area of  .

**7c.** *[8 marks]*

Consider all values of  such that the graphs of  and  intersect. Find the value of  that gives the greatest value for the area of  .

**8a.** *[1 mark]*

Write down the value of

(i) ;

**8b.** *[1 mark]*

(ii) ;

**8c.** *[1 mark]*

(iii) .

**8d.** *[3 marks]*

Hence, solve .

**9a.** *[2 marks]*

Find the value of each of the following, giving your answer as an integer.



**9b.** *[2 marks]*



**9c.** *[3 marks]*



**10a.** *[2 marks]*

The number of bacteria in two colonies,  and , starts increasing at the same time.

The number of bacteria in colony  after  hours is modelled by the function .

Find the number of bacteria in colony  after four hours.

**10b.** *[3 marks]*

Find the number of bacteria in colony  after four hours.

**10c.** *[3 marks]*

How long does it take for the number of bacteria in colony  to reach ?

**10d.** *[3 marks]*

The number of bacteria in colony  after  hours is modelled by the function .

After four hours, there are  bacteria in colony . Find the value of .

**10e.** *[4 marks]*

The number of bacteria in colony  after  hours is modelled by the function .

The number of bacteria in colony  first exceeds the number of bacteria in colony  after  hours, where . Find the value of .

Printed for British School of Beijing

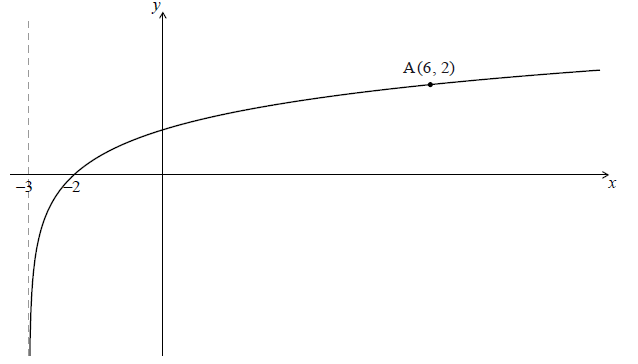
© International Baccalaureate Organization 2015

International Baccalaureate® - Baccalauréat International® - Bachillerato Internacional®

# Chp 3 & 4 Questions

**1a.** *[4 marks]*

Let  for  . Part of the graph of *f* is shown below.



The graph passes through A(6, 2) , has an *x*-intercept at (−2, 0) and has an asymptote at  .

## Markscheme

evidence of substituting the point A ***(M1)***

e.g. 

manipulating logs ***A1***

e.g. 

***A2 N2***

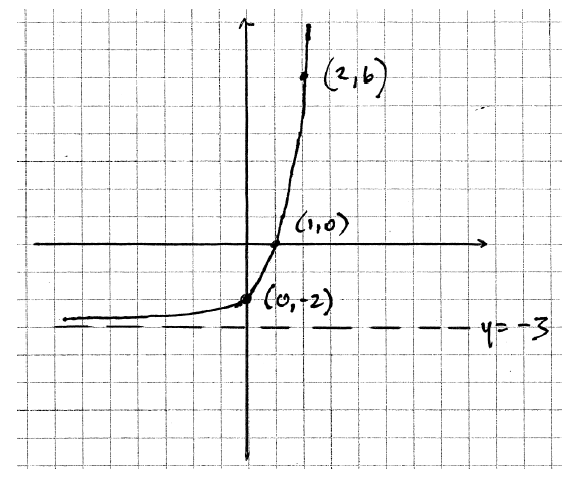
***[4 marks]***

**1b.** *[5 marks]*

## Markscheme

(i)  (accept ***A1 N1***

(ii)

 ***A1A1A1A1 N4***

**Note**: Award ***A1*** for asymptote at  , ***A1*** for an increasing function that is concave up, ***A1*** for a positive *x*-intercept and a negative *y*-intercept, ***A1*** for passing through the point  .

***[5 marks]***

**1c.** *[4 marks]*

## Markscheme

**METHOD 1**

recognizing that ***(R1)***

evidence of valid approach ***(M1)***

e.g. switching *x* and *y* (seen anywhere), solving for *x*

correct manipulation ***(A1)***

e.g. 

***A1 N3***

**METHOD 2**

recognizing that ***(R1)***

identifying vertical translation ***(A1)***

e.g. graph shifted down 3 units, 

evidence of valid approach ***(M1)***

e.g. substituting point to identify the base

***A1 N3***

***[4 marks]***

**2a.** *[1 mark]*

Jose takes medication. After *t* minutes, the concentration of medication left in his bloodstream is given by  , where *A* is in milligrams per litre.

## Markscheme

***A1 N1***

***[1 mark]***

**2b.** *[2 marks]*

## Markscheme

substitution into formula ***(A1)***

e.g.  , 

***A1 N2***

***[2 marks]***

**2c.** *[5 marks]*

## Markscheme

set up equation ***(M1)***

e.g. 

attempting to solve  ***(M1)***

e.g. graph, use of logs

correct working ***(A1)***

e.g. sketch of intersection, 

***A1***

correct time 18:33 or 18:34 (accept 6:33 or 6:34 but nothing else) ***A1 N3***

***[5 marks]***

**3a.** *[2 marks]*

Let  , for  .

## Markscheme

combining 2 terms ***(A1)***

e.g.  , 

expression which clearly leads to answer given ***A1***

e.g.  , 

***AG N0***

***[2 marks]***

**3b.** *[3 marks]*

## Markscheme

attempt to substitute either value into *f*  ***(M1)***

e.g.  , 

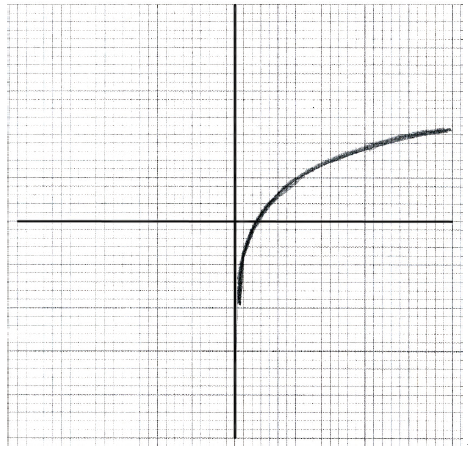
 , ***A1A1 N3***

***[3 marks]***

**3c.** *[6 marks]*

## Markscheme

(i)  , ***A1A1 N1N1***

(ii) ***A1A1A1 N3***

**Note**: Award ***A1*** for sketch approximately through  , ***A1*** for approximately correct shape, ***A1*** for sketch asymptotic to the *y*-axis.

(iii)  (must be an equation) ***A1 N1***

***[6 marks]***

**3d.** *[1 mark]*

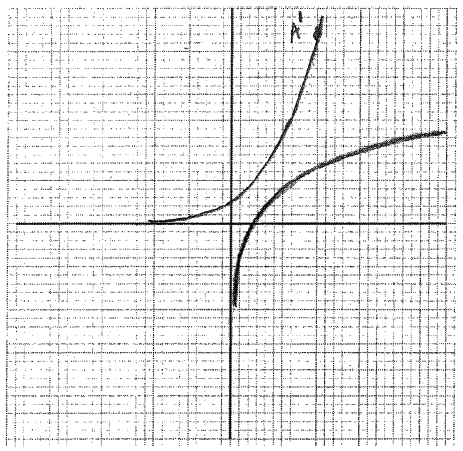
## Markscheme

***A1 N1***

***[1 mark]***

**3e.** *[4 marks]*

## Markscheme

 ***A1A1A1A1 N4***

**Note**: Award ***A1*** for sketch approximately through  , ***A1*** for approximately correct shape of the graph reflected over  , ***A1*** for sketch asymptotic to *x*-axis, ***A1*** for point  clearly marked and on curve.

***[4 marks]***

**4a.** *[4 marks]*

Let  and  .

## Markscheme

attempt to apply rules of logarithms ***(M1)***

e.g.  , 

correct application of  (seen anywhere) ***A1***

e.g. 

correct application of  (seen anywhere) ***A1***

e.g. 

so 

 (accept  ) ***A1 N1***

***[4 marks]***

**4b.** *[3 marks]*

## Markscheme

transformation with correct name, direction, and value ***A3***

e.g. translation by , shift up by  , vertical translation of 

***[3 marks]***

**5a.** *[3 marks]*

## Markscheme

evidence of correct formula ***(M1)***

*eg*  ,  , 

**Note:** Ignore missing or incorrect base.

correct working ***(A1)***

*eg*   , 

***A1 N2***

***[3 marks]***

**5b.** *[4 marks]*

## Markscheme

attempt to write  as a power of  (seen anywhere)  ***(M1)***

*eg*  ,  , 

multiplying powers  ***(M1)***

*eg*  , 

correct working  ***(A1)***

*eg*  ,  , 

***A1 N3***

***[4 marks]***

**6a.** *[7 marks]*

Let  and  .

## Markscheme

(a) **METHOD 1**

evidence of correct formula ***(M1)***

*eg*  , 

***A1 N2***

**METHOD 2**

valid method using ***(M1)***

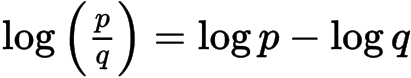
*eg*  ,  , 

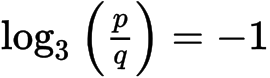
***A1 N2***

***[2 marks]***

(b) **METHOD 1**

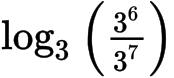
evidence of correct formula ***(M1)***

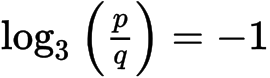
*eg*  , 

***A1 N2***

**METHOD 2**

valid method using  and ***(M1)***

*eg*  ,  , 

***A1 N2***

***[2 marks]***

(c) **METHOD 1**

evidence of correct formula ***(M1)***

*eg*  , 

 (may be seen in expression)  ***A1***

*eg* 

***A1 N2***

**METHOD 2**

valid method using ***(M1)***

*eg*  , 

correct working ***A1***

*eg*  , 

***A1 N2***

***[3 marks]***

***Total [7 marks]***

**6b.** *[2 marks]*

## Markscheme

**METHOD 1**

evidence of correct formula ***(M1)***

*eg*  , 

***A1 N2***

**METHOD 2**

valid method using ***(M1)***

*eg*  ,  , 

***A1 N2***

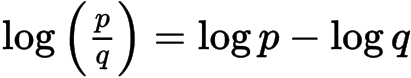
***[2 marks]***

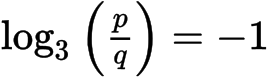
**6c.** *[2 marks]*

## Markscheme

**METHOD 1**

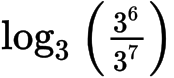
evidence of correct formula ***(M1)***

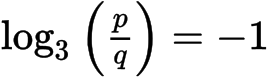
*eg*  , 

***A1 N2***

**METHOD 2**

valid method using  and ***(M1)***

*eg*  ,  , 

***A1 N2***

***[2 marks]***

**6d.** *[3 marks]*

## Markscheme

**METHOD 1**

evidence of correct formula ***(M1)***

*eg*  , 

 (may be seen in expression)  ***A1***

*eg* 

***A1 N2***

**METHOD 2**

valid method using ***(M1)***

*eg*  , 

correct working ***A1***

*eg*  , 

***A1 N2***

***[3 marks]***

***Total [7 marks]***

**7a.** *[7 marks]*

Let  and  , where  , and  . Let  be the region enclosed by the -axis, the graph of  , and the graph of  .

Let .

## Markscheme

***(i)***

***A1A1 N2***

**Notes**: Award ***A1*** for the graph of  positive, increasing and concave up.

Award ***A1*** for graph of  increasing and linear with -intercept of .

Penalize one mark if domain is not [, ] and/or if  and  do not intersect in the first quadrant.

***[2 marks]***

***(ii)attempt to find intersection of the graphs of***  ***and***  ***(M1)eg***  ***A1valid attempt to find area of***  ***(M1)eg***  ***,***  ***,*** ***area***  ***A2 N3[5 marks]***

**7b.** *[5 marks]*

## Markscheme

attempt to find intersection of the graphs of  and  ***(M1)***

*eg*  

***A1***

valid attempt to find area of ***(M1)***

*eg*   ,  , 

area ***A2 N3***

***[5 marks]***

**7c.** *[8 marks]*

## Markscheme

recognize that area of  is a maximum at point of tangency ***(R1)***

*eg*  

equating functions  ***(M1)***

*eg*   , 

 ***(A1)***

equating gradients ***(A1)***

*eg*   , 

attempt to solve system of two equations for ***(M1)***

*eg*  

***(A1)***

attempt to find ***(M1)***

*eg*   , 

 (exact), ***A1 N3***

***[8 marks]***

**8a.** *[1 mark]*

Write down the value of

## Markscheme

(i) ***A1 N1***

***[1 mark]***

**8b.** *[1 mark]*

## Markscheme

(ii) ***A1 N1***

***[1 mark]***

**8c.** *[1 mark]*

## Markscheme

(iii) ***A1 N1***

***[1 mark]***

**8d.** *[3 marks]*

## Markscheme

correct equation with **their** three values ***(A1)***

*eg* 

correct working involving powers ***(A1)***

*eg* 

***A1 N2***

***[3 marks]***

**9a.** *[2 marks]*

Find the value of each of the following, giving your answer as an integer.

## Markscheme

correct approach ***(A1)***

*eg* 

***A1 N2***

***[2 marks]***

**9b.** *[2 marks]*

## Markscheme

correct simplification ***(A1)***

*eg* 

***A1 N2***

***[2 marks]***

**9c.** *[3 marks]*

## Markscheme

correct simplification ***(A1)***

*eg* 

correct working ***(A1)***

*eg* 

***A1 N2***

***[3 marks]***

**10a.** *[2 marks]*

The number of bacteria in two colonies,  and , starts increasing at the same time.

The number of bacteria in colony  after  hours is modelled by the function .

## Markscheme

correct substitution into formula ***(A1)***

*eg* 

 bacteria in the dish ***A1 N2***

***[2 marks]***

**10b.** *[3 marks]*

## Markscheme

correct substitution into formula ***(A1)***

*eg* 

***(A1)***

 bacteria in the dish (integer answer only) ***A1 N3***

***[3 marks]***

**10c.** *[3 marks]*

## Markscheme

correct equation ***(A1)***

*eg* 

valid attempt to solve ***(M1)***

*eg* graph, use of logs



 (hours) ***A1 N3***

***[3 marks]***

**10d.** *[3 marks]*

## Markscheme

valid attempt to solve ***(M1)***

*eg* , use of logs

correct working ***(A1)***

*eg* sketch of intersection, 



 (exact), ***A1 N3***

***[3 marks]***

**10e.** *[4 marks]*

## Markscheme

**METHOD 1**

setting up an equation or inequality (accept any variable for ) ***(M1)***

*eg* 

correct working ***(A1)***

*eg* sketch of intersection, 

 (accept ) ***(A1)***

 (integer answer only) ***A1 N3***

**METHOD 2**

 (from earlier work)

***A1A1***

valid reasoning ***(R1)***

*eg* **and** 

 (integer answer only) ***A1 N3***

***[4 marks]***

Printed for British School of Beijing

© International Baccalaureate Organization 2015

International Baccalaureate® - Baccalauréat International® - Bachillerato Internacional®