The first two thousand and ten decimal places of π

π = 3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679
8214808651 3282306647 0938446095 5058223172 5359408128
4811174502 8410270193 8521105559 6446229489 5493038196
4428810975 6659334461 2847564823 3786783165 2712019091

4564856692 3460348610 4543266482 1339360726 0249141273
7245870066 0631558817 4881520920 9628292540 9171536436
7892590360 0113305305 4882046652 1384146951 9415116094
3305727036 5759591953 0921861173 8193261179 3105118548
0744623799 6274956735 1885752724 8912279381 8301194912

9833673362 4406566430 8602139494 6395224737 1907021798
6094370277 0539217176 2931767523 8467481846 7669405132
0005681271 4526356082 7785771342 7577896091 7363717872
1468440901 2249534301 4654958537 1050792279 6892589235
4201995611 2129021960 8640344181 5981362977 4771309960

5187072113 4999999837 2978049951 0597317328 1609631859
5024459455 3469083026 4252230825 3344685035 2619311881
7101000313 7838752886 5875332083 8142061717 7669147303
5982534904 2875546873 1159562863 8823537875 9375195778
1857780532 1712268066 1300192787 6611195909 2164201989

3809525720 1065485863 2788659361 5338182796 8230301952
0353018529 6899577362 2599413891 2497217752 8347913151
5574857242 4541506959 5082953311 6861727855 8890750983
8175463746 4939319255 0604009277 0167113900 9848824012
8583616035 6370766010 4710181942 9555961989 4676783744

9448255379 7747268471 0404753464 6208046684 2590694912
9331367702 8989152104 7521620569 6602405803 8150193511
2533824300 3558764024 7496473263 9141992726 0426992279
6782354781 6360093417 2164121992 4586315030 2861829745
5570674983 8505494588 5869269956 9092721079 7509302955

3211653449 8720275596 0236480665 4991198818 3479775356
6369807426 5425278625 5181841757 4672890977 7727938000
8164706001 6145249192 1732172147 7235014144 1973568548
1613611573 5255213347 5741849468 4385233239 0739414333
4547762416 8625189835 6948556209 9219222184 2725502542

5688767179 0494601653 4668049886 2723279178 6085784383
8279679766 8145410095 3883786360 9506800642 2512520511
7392984896 0841284886 2694560424 1965285022 2106611863
0674427862 2039194945 0471237137 8696095636 4371917287
4677646575 7396241389 0865832645 9958133904 7802759009

9465764078 ...

12 Functions

The activity explores the mathematical properties of a linear function, continuing the multiplicative relations theme of *TM8: Relations*.

Proportionality is introduced implicitly.

Thinking strands
Ratio and proportion

Curriculum links
Multiplicative relation
Graphs

Resources
Notesheets 1 to 3

Lesson summary

1 Introduction to Notesheet 1, class discussion (15 min)

Give out Notesheet 1. Write the ordered pairs of values from the Notesheet on the board. Explain these can be looked at in terms of an input variable – something you change – and an output variable – something you measure after fixing the input. Ask pupils how they can construct a table to help them make sense of the relation between the values mathematically. Try any ideas offered on the board. When the idea of listing them in numerical order comes up and seems to them a good strategy, give them just 5 minutes to do Notesheet 1.

2 Coordinating two relations, Notesheet 2 (20 min)

As introduction record on the board the pupils' answers to question 3 on Notesheet 1. Say their task is to find examples and pairs of values in their table for question 1, which will enable them to answer in words question 2 on the Notesheet. Pupils should work in small groups; then in class discussion write down on the board all their answers for question 2.

3 Teaching episode (5 min)

Explain that a relationship like this is called a **function**. Write the Input and Output values from Notesheet 1 in two parallel ordered sets on the board, and point out

(a) there is only one Output value to each Input

(b) as the Input goes up, so does the Output.

Point out that in their answers to question 2 on Notesheet 2 they have said more about this function.

4 Notesheet 3 (15 min)

Pupils plot the graph of Output against Input. Class discussion should feature:

(a) graph goes up at the same **rate** throughout (the linear aspect of it)

(b) all values of the Output are 5 times the values of the corresponding Input.

Pupils then predict Output values from given Inputs. In question 4 they find some way of expressing this function in terms of generalised number.

If any time remains, ask the pupils what they think their results mean in terms of the cogs and the number of teeth in each.

Mathematical content

Pupils find a multiplicative relation that underlies a function, and draw Cartesian graphs of it. As with much work in science at this level, the focus is on monotonic functions.

In science pupils are often given an experiment where the values that they obtain and then plot on a graph – for example, the stretching of a spring under an increasing load – fit a linear functional relationship of the form $y = mx$. The straight line graph then gives an implicit model that a simple causal relationship exists between the variables, but the mathematical properties of the relation are rarely explored explicitly. In mathematics the treatment of a linear function and its graphical expression may come much later. This may be one of the reasons why pupils fail to link what they do in science and what they do in mathematics.

There are two multiplicative relations involved in a linear function such as $y = mx$ (this distinction goes back to Aristotle, as the two ways of looking at a proportion – *within* and *between*). Within the values of Input and Output is the constant ratio of m. Between the values, any proportional change you make to one value

(for example, making it three times bigger) makes the same proportional change in the other. The second aspect you need in science, and the first you need in maths, but the understanding of proportion only comes from integrating the two.

Pupils' thinking

This activity is one of the first of Phase II of the project where, from a Piagetian point of view, although most of the activity can be processed using concrete operational thinking, we are now definitely pushing towards formal thinking. In this case that means construing the relation between the numbers in multiplicative, ratio, terms. (See the two examples given in 'The thinking behind *Thinking Maths*' in the Introduction.)

The generalised number aspect of the activity is aimed ahead at making pupils more familiar with some of the language of algebra.

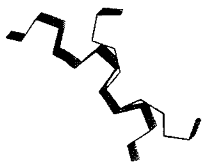
The focus of this lesson is on generating many-layered pupil discussion. The more ways of looking at the numbers that the pupils come up with, the more will each pupil be helped in his/her thinking at the point where they currently are. The steepness of the graph is not mentioned, but some pupils may want to look at this as expressing how much bigger the values of one variable are compared with the other.

Specimen lesson

This is a class with a low attaining profile of ability. The class teacher is supported by a Special Needs support teacher (SNT).

0.00 Notesheet 1.

On the board she draws:



"Who has seen cogs turning together?"

Pupils:
"In a car."
"In a windmill."
"On bikes."

Teacher: "Which cog will turn more?"

Pupils: "Little one will go round quicker ..."

Teacher: "Are you sure that it was the small one that went round more times?"

Boy replies: "Yes, say the big one's gone round half, then . . ." (can't finish explanation)

Girl adds: "With the smaller one, it has a smaller distance to travel until it's gone all the way round."

Teacher: "With the table underneath, have you met Input/Output before?"

Pupils:
"In Science."
"Put 8 in Input and 40 in Output."
Boy starts: "Because you put ..."
Another completes: "With a computer, it's the same."

Teacher: "Can you give some examples?"

Pupils:
"How much energy you put in; how much energy you get out."
"Fuel in – exhaust out."

Teacher: "Or petrol in, and how many miles you go."

"Input, in Science, is something we have control of. So now put the numbers into the table in a sensible way."

Five or six in the class spontaneously order by the Input variable in ascending or descending order (one or two with help).

Teacher prepares table on board, takes entries from the pupils.

Input	Output
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40

Teacher: "What do you notice?"

Pupils:
"They are all times-across."
"They make the 5 times table."
"They are like the clock times."

Teacher: "Could they get too big for the clock?"

Pupil: "Yes, if you had 20."

Teacher: "If you went on with the table, when would you stop?"

Pupils:
"Never!"
"Go on forever."
Teacher, pointing to Input column: "What's happening as we go down?"

Pupil: "Up in ones."

Teacher: "Suppose we had 20?"

Pupils look mystified.

Teacher: "I mean in the Input."

Pupils: "Count down in 5s."
(meaning iteration until 20 reached)

Teacher: "If you had a calculator?"

Pupils:
"120."
"Multiply 5 by 100."
"Times 5 by big wheel number."
" 1 × 5; 2 × 5; 3 × 5 etc."
(Pupils still using iteration.)

Teacher: "So, is 20 times 5, 100?"

Pupils:
" Yes."
"No."
"Yes."
"Yes."

0.15 Notesheet 2

The teacher gives out her own Notesheet 2, with doubling and tripling strategy shown on it by arrowed loops from number to number. The list is extended to an Input of 10.

"Do you understand this?"

Pupils: "You've changed it!"

The teacher adds linkages to table on the board as well, explaining the Notesheet.

"Doubling it, or timesing it by two."

Pupils: "You added it."

Teacher: "Yes, but doubling's a better word."

During their work on the task nearly all the pupils come out with 'same' for their written answers about what they had noticed. When pressed they can (with reluctance) expand it and say what was the same in both, but it is as if the effort to see the sameness is quite enough, and the extra effort of expressing it verbally as well is too much.

0.25 Class discussion

Teacher: "Some people have done their own table. Some did some new ones."

The teacher shows a curved arrow from 4 to 8 in Input.

"If the arrow this way means doubling, what would it mean if the direction of the arrow were the other way?"

Pupils: "It would mean halving."

Teacher shows it: "Arrow other way is half". She asks for other values.

Pupil: "3 and 6."
"So what happened?"
"We doubled."

Teacher: "And what happened on this side? (Output)"

Pupil: "It was doubled also."

Teacher: "If we go from top to bottom of table, how many times? – JB?"

Pupil: "Ten."

"If your theory is correct this one should be...?"
"50."
"Was that multiplying by 10 also?"
"Yes."
"It doesn't have to be something easy like doubling – the method also works for timesing by 10."

0.30 Notesheet 3

"Now we'll try to graph that." (meaning the doubling table).

The teacher gives out her own Notesheet 3, which has a graph with only part of each axis with numbers entered. Pupils are asked to make the input axis go up to 50 (from 40) and the output go to 80 (from 40 also). Units have to be interpolated by eye with only the 5s marked.

During work on the task, 7 pupils use dots hesitantly for entering values; 3 or 4 still trying bars, until SNT shows how to plot points. Even the weakest pupils had been asked to do this before in Science (but did they manage?).

One pupil has a point off, but is able to self-correct when he is shown it doesn't fit the pattern of the other points.

0.35 Class discussion

"Tell me what you noticed about the points?"

Pupils:
"Went up in 1s."
"No ... 5s."
"When you connect them up they went in a straight line."

Teacher: "Carl said something about the graph ... ?"

Pupils: "It could go on forever."

Girl adds: "You can't get all the paper in the world!"

"Input was 5 times output...No! output was 5 times input."

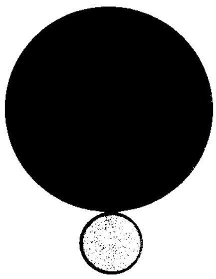
Teacher now records in summary on board:
Table goes on for ever
Line goes on for ever
Output is 5 times Input

Teacher: "We get the same times bigger as we move down each side of the table" (shows).

"Imagine if we made the wheel bigger. What difference would that make?"

Pupils:
"Depends on which wheel is made bigger."
"Numbers on the other wheel would get bigger."
"And on how much bigger."

Teacher shows on board a differently scaled pair of cog-wheels.



"Would the times number be larger or smaller?"

Pupils: "Bigger."

Teacher: "If we moved down one side of the (new) table by doubling..?"

Pupil: "Would still be the same on the other."

Teacher: "Other side doubled also."

0.50

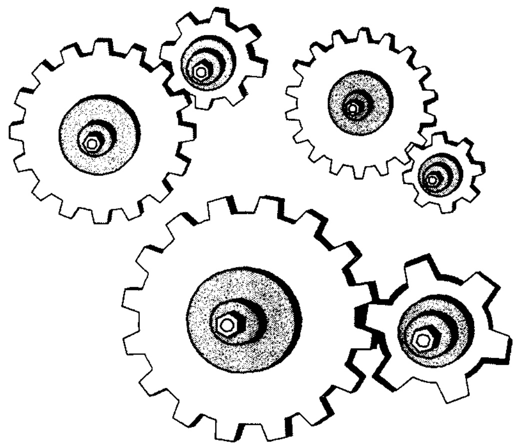
Teacher congratulates pupils on the contribution they have made to the lesson.

Lesson ends.

Input and Output

In a Science lesson pupils were given two cog-wheels which meshed into each other.

The larger wheel had a handle.



The pupils counted how many full turns the **smaller cog** made each time the **larger one** turned a number of full turns. For example, when the large cog turned 8 times, the small cog turned 40 times.

Here are the results from 8 different groups of pupils. In each pair the number of turns of the large cog is written first.

8, 40 3, 15 6, 30 2, 10 5, 25 7, 35 1, 5 4, 20

1 How to make sense of these results mathematically?
Think of a sensible way to arrange the values in this table:

Input	Output

2 What idea did you use in filling in the table?

3 What pattern(s) can you see in the results?

12 Functions

Notesheet 2

Relation between Input and Output

1 Use your table to find **examples** for these questions:

(a) If you **double** (×2) the Input value, how many times does the Output value go up? Find 3 examples from your table.

(b) If you **triple** (×3) the Input value, how many times does the Output value go up? Find 2 examples from your table.

(c) If you make the Input value **four times smaller (÷4)**, how many times smaller does the Output value become? Find two examples from your table.

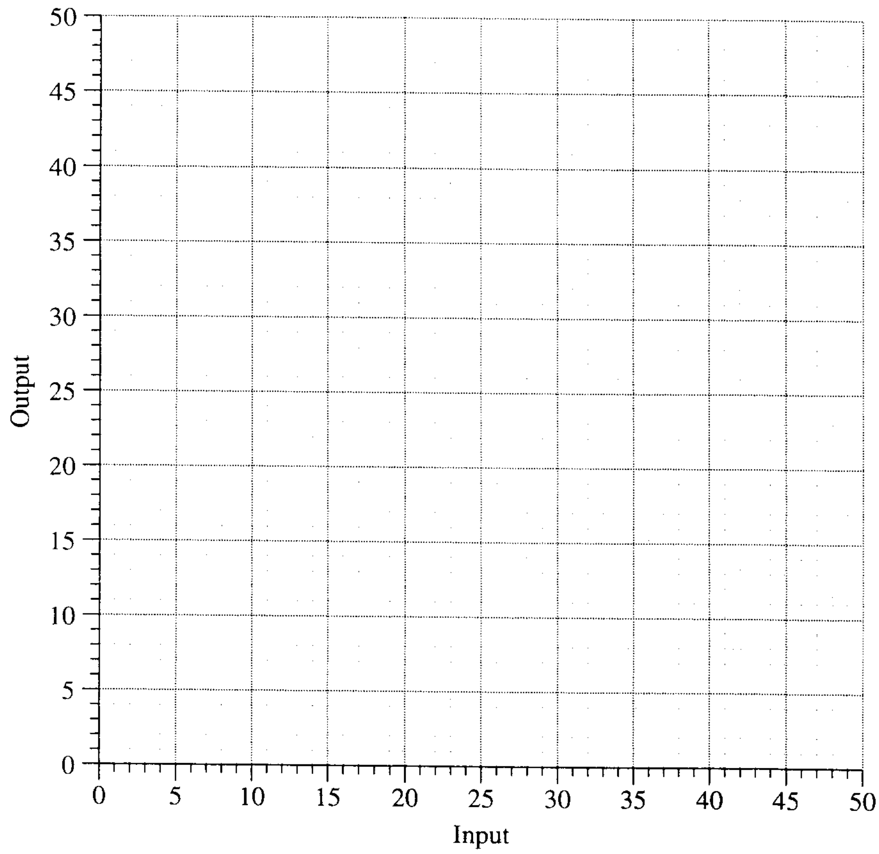
2 Now use your answers to question 1 and discuss with your partner(s) how to answer this question in words:
'What is the general rule which relates a change you make to the Input variable and the change which occurs in the Output variable?'

3 If you made a graph of the results in your table, what shape do you think it would be? (Don't draw the graph!)

Graph of the relation

On this graph grid the horizontal axis is for Input, and the vertical axis is for Output.

1 Plot the 8 results from your table on the graph.



2 In words, what is the relation between the Input and the Output?
(Between the number of turns of the large cog-wheel and number of turns of the small wheel.)

3 With your partner(s) find two ways – one using the graph and one using calculation – to predict the Output value when the Input value is 10.
Decide which of these you have to use if the input value is 20.

4 Now use x to stand for the Input, and y for the Output and write an equation for the relationship.

6 Direction and distance Notesheet 1

Compass robot

Write a program to send the robot to drop food at G, D, E and F in that order.
Use N, S, E, W and distance, where the distance is the number of steps on the grid lines.

Program

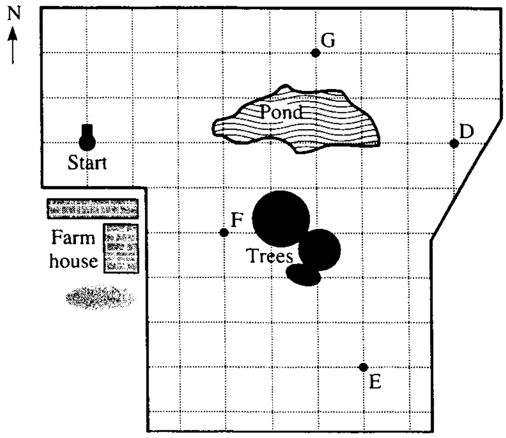
From
Start to ... : ...., ..., Drop
(direction and distance) (direction and distance)

... to ... : ...., ..., Drop

... to ... : ...., ..., Drop

... to ... : ...., ..., Drop

... to ... : ...., ...



1 Write the move from Start to G. ......, ...

Write the move back, the reverse. ......, ...

2 Explain how to find the reverse of any move without using the grid.

Left/right robot

Write the program for this robot. Use Forward, Backward, Right, Left, Distance.

Program

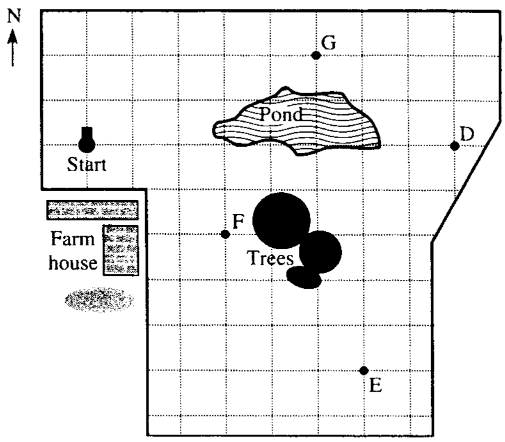
From
Start to ... : ...., ..., Drop
(direction and distance) (direction and distance)

... to ... : ...., ..., Drop

... to ... : ...., ..., Drop

... to ... : ...., ..., Drop

... to ... : ...., ...



3 The farmer prefers the Compass robot. Give a possible reason:

4 If someone asks you the way in the street, which method do you use? Why?

6 Direction and distance Notesheet 2

Angle compass robot

Write the program. Use angle clockwise from North, distance in cm.

Program

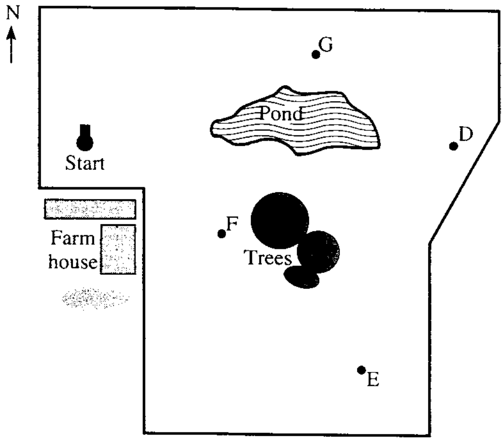
From
Start to ... : ...., Drop
(angle–distance)

... to ... : ...., Drop

... to ... : ...., Drop

... to ... : ...., Drop

... to ... : ....



1 List the advantages of this robot over the previous two robots.

2 List the disadvantages.

Left & right angle robot

Write the program. Use Right or Left and Angle then Distance in cm. Right, or left, is the way you want to turn and the angle is measured from the line you have just travelled.

Program

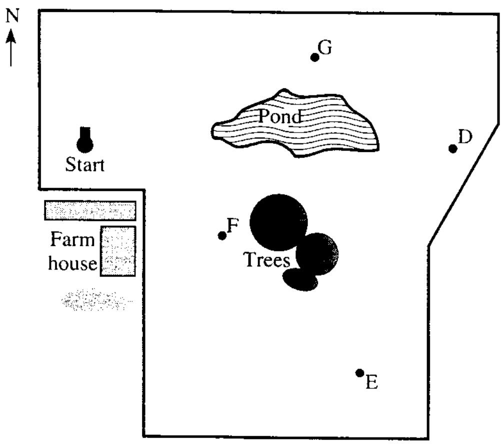
From
Start to ... : ...., Drop
(R/L angle–distance)

... to ... : ...., Drop

... to ... : ...., Drop

... to ... : ...., Drop

... to ... : ....



3 Think of the four methods of giving direction you have used in Notesheets 1 and 2. Which would you use for:

(a) Navigation at sea ....

(b) Giving someone directions from school to the station ....

(c) Orienteering in the mountains ....

(d) Giving directions in a block of flats ....

(e) A treasure hunt game in open grounds ....